Tropospheric Propagation Modeling in the Modern Air Defense Environment

Th. Papastamatis[a], K. Ioannou[b], I. Koukos[c] and E. Papageorgiou[d]

[a]*Hellenic Air Force, Lieutenant & PhD Candidate, Aristotle University of Thessaloniki, Greece*

[b]*Hellenic Navy, Lieutenant Junior Grade & PhD Candidate, Aristotle University of Thessaloniki, Greece*

[c] *Combat Systems Sector, Hellenic Naval Academy, Piraeus, Greece*

[d] *Mathematics Sector, Hellenic Naval Academy, Piraeus, Greece*

Abstract. The threat faced by a modern Air Defense System has become very diverse. Traditional threats of bomber and fighter-bomber aircraft remain, but more nations are obtaining access to ballistic missiles and cruise missiles, which could attack from unexpected directions and in large numbers, saturating defenses. Advanced technologies for stealth and navigation are making the threat from conventional weapon systems even greater. Modern Air Defense Systems in the era of stealth aircraft become increasingly sophisticated in order to exploit the geographic aspect of the radiocoverage of the wide theater of operations. The monostatic radar of the 20th century will be replaced rapidly by hybrid multistatic clusters involving active and passive sensing of a multimode network centric Air Defense Model. Therefore the classic tropospheric propagation modeling has to be reexamined in order to optimize the radiocoverage of the defense networks that are confronted by stealthy aircrafts along with the weapons delivered. This paper reviews the advantages and the difficulties of safeguarding the effective operation of a defense network from an electromagnetic point of view.

Keywords: Refraction effects prediction, radar radio coverage.

1. INTRODUCTION

Refraction is an omnipresent natural phenomenon occurring wherever there is a transmission medium of variable density which affects the velocity of a wave propagation. In the troposphere, the lowest part of the atmosphere named from the existence of weather phenomena which extend up to 10 km, the wireless transmission of signals either from radar or telecom transceivers is affected by refraction due to changes in the refractive index n that depends on the air temperature, pressure and humidity as shown in the ITU RECOMMENDATION ITU-R P.453-9 [5].

The refractive index n is defined as the ratio of the speed of radio waves in vacuum to the speed in the medium under consideration. The atmospheric radio refractive index, n, can be computed by the following formula**:**

$$n = 1 + N \ x \ 10^{-6} \tag{1}$$

where N is the radio refractivity expressed by:

$$N = N_{dry} + N_{wet} = \frac{77{,}6}{T}(P + 4810\frac{e}{T}) \tag{2}$$

The dry component N_{dry} of radio refractivity is given by

$$N_{dry} = 77{,}6\frac{P}{T} \tag{3}$$

and the wet component

$$N_{wet} = 3{,}732 \times 105\frac{e}{T^2} \tag{4}$$

where P is the atmospheric pressure (hPa), e is the water vapor pressure (hPa) and T is the absolute temperature (K). This expressions for frequencies up to 100 GHz contain error less than 0,5%, therefore are acceptable for use.

An atmosphere having a standard refractivity gradient $\frac{dN}{dh} = -40$, the effective radius of the Earth is about $4/3$ that of the actual radius, which corresponds to approximately 8 500 km and is called commonly as the “$4/3$ earth approximation”.

The Standard radio atmosphere is defined as an atmosphere having the standard refractivity gradient. The refractivity gradient variations from the standard value -40 correspond to significant refractive phenomena severely affecting the electromagnetic ray paths. Thus we call **Sub-refraction** the refraction for which the refractivity gradient is greater (i.e. positive or less negative) than the standard refractivity gradient. Similarly we call **Super-refraction** the refraction for which the refractivity gradient is less (i.e. more negative) than the standard refractivity gradient.

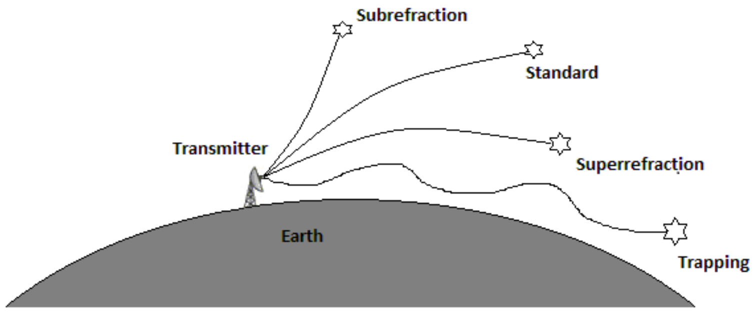

Figure 1. Types of refraction compared to the standard refraction with gradient = -40 corresponding to the 4/3 Earth Approximation

The trapping or ducting phenomenon occurs when overlapping atmospheric slabs rather reflect than refract rays ie. angle of refraction$a_i \leq 0^0$, because of abrupt temperature and pressure variations. Thus a ray is captured within a long path forming a parallel plate waveguide above the earth's surface that carries the electromagnetic wave far beyond line-of-sight distances. A duct is formed when temperature increases near surface abruptly (temperature inversion) or the humidity decreases (moisture lapse) with height.

A radar processor measures target ranges assuming straight line ray propagation. Because of ray bending that is not a correct assumption, thus radar senses only an apparent range and an apparent angle of elevation that differ from their true values as indicated in the Figure 2 below:

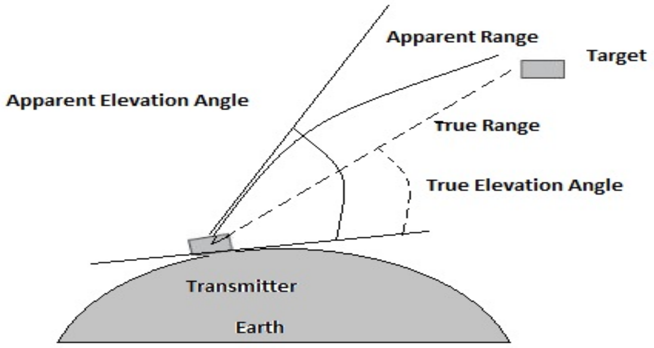

Figure 2. Apparent Target Range and Angle of Elevation due to ray bending as compared to True Values assuming (nonexistent) straight ray propagation

The calculation of the induced range and elevation errors is depicted in Figure 3.

ISSN:1791-4469

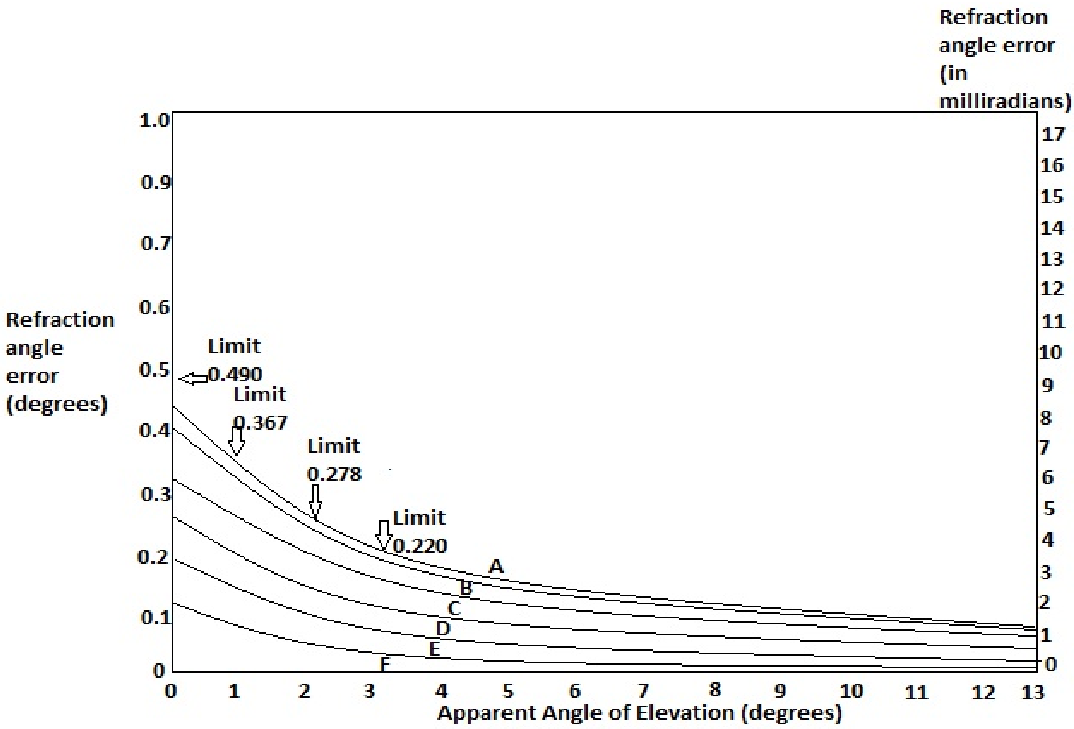

Figure 3. Normal refraction errors in range (right axis) and angle (left axis) vs apparent angle of elevation

In the ITU-R P.453-9 recommendation the long-term mean dependence of the refractive index n upon the height h is well expressed by an exponential law:

$$n(h) \;=\; 1 + N_0 \times 10^{-6} \times e^{-\frac{h}{h_0}} \tag{5}$$

where, N_0 is the average value of atmospheric refractivity extrapolated to sea level and h_0 is the scale height (km). N_0 and h_0 can be determined statistically for different climates. For reference purposes a global mean of the height profile of refractivity may be defined by: $N_0 = 315$ and $h_0 = 7.35$ Km.These numerical values apply only for terrestrial paths. This reference profile may be used to compute the value of refractivity N_s at the Earth's surface from N_0 as follows:

$$N_s = N_0 \times e^{-\frac{h_s}{h_0}} \tag{6}$$

Where, h_s= height of the Earth's surface above sea level (km).
In order to examine the N gradients, the modified refractivity index issued. It is defined as:

$$M = \left(n - 1 + \frac{h}{R_e}\right) \times 10^6 = N + 0{,}157 \tag{7}$$

The computation of the refractive conditions, characterized as Subrefraction, Standard, Superrefraction and Trapping is achieved by its gradient $\frac{dM}{dh}$ as shown in the table below. Tropospheric ducting phenomena occur when either inequality $\frac{dM}{dh} < 0$ or $\frac{dN}{dh} < -157$ is satisfied.

Refraction	**Sub-refraction**	**Standard**	**Super-refraction**	**Trapping**
$\frac{dM}{dh}, \frac{M-Units}{km}$	$\frac{dM}{dh} < 157$	$78 < \frac{dM}{dh} \leq 157$	$0 < \frac{dM}{dh} \leq 78$	$\frac{dM}{dh} \leq 0$
$\frac{dN}{dh}, \frac{N-Units}{km}$	$\frac{dN}{dh} > 0$	$-79 < \frac{dN}{dh} \leq 0$	$-157 < \frac{dN}{dh} \leq -79$	$\frac{dN}{dh} \leq -157$

Table 1. Computation of the refractive conditions, characterized as Subrefraction, Standard, Super-refraction and Trapping

Of great importance for radar propagation near sea surface are the evaporation ducts. An evaporation duct is a tropospheric phenomenon that primarily occurs immediately above the surface of the sea and other large bodies of water and exists because the amount of water vapor present in the air decreases rapidly with height in the first few meters above the surface of the water. By virtue of its nature of formation, an evaporation duct is a nearly permanent feature. The height of an evaporation duct is typically of the order of only a few meters (the world average evaporation duct height is reported to be approximately 13 m, however this can vary considerably with geographical location and changes in atmospheric parameters such as humidity, temperature and wind speed. The evaporation duct height is often used as an indicator of its strength and its ability to trap radio waves [2].

2. A DIFFERENT APPROACH OF MODELING THE REFRACTION INDEX VARIATIONS ON RADIOWAVE PROPAGATION

It has been observed that in the troposphere, the relative dielectric constant is slightly higher than unity due to the presence of the atmosphere and, in particular, water vapor. The tropospheric region extends from the surface of the earth to a height of about 6 km at the poles and 18 km at the equator. The relative dielectric constant is a function of the temperature, pressure, and humidity (or water vapor pressure). The typical value of ε_r at the surface of the earth is found to be 1.000579 [1]. The value decreases as a function of height above the surface of the earth.

ISSN:1791-4469

$$v = \frac{c}{\sqrt{\varepsilon_r}} = \frac{c}{n} \tag{8}$$

where c is the velocity in vacuum and $n = \sqrt{\varepsilon_r}$ is the refractive index of the medium. At the surface of the earth (mean sea level), the refractive index of air is 1.000289. Therefore, it is common practice to work with a parameter known as the refractivity, N. The refractivity is related to refractive index by the following equation.

$$N = (n - 1) \times 10^6 \tag{9}$$

Thus, at the surface of the earth, the refractivity is equal to 289. For a standard atmosphere, the refractivity falls off linearly up to a height of 2 km above the surface of the earth and is expressed by the following equation

$$N = 289 - 39h \tag{10}$$

where h is the height in km. This is known as the standard atmosphere or normal atmosphere. The refractive index of the standard atmosphere is given by

$$n = 1 + (289 - 39h) \times 10^6 \tag{11}$$

Let us now derive an equation for the path of a wave propagating in a medium wherein the refractive index is a continuous function of height. For this derivation, let us assume that the earth is flat and the troposphere is made up of stratified layers parallel to the surface of the earth, i.e., the refractive index is a function of height only. Let the refractive index be a constant within each layer. Consider a layer that has a height dh with refractive index $(n + dn)$ (Figure 4). A ray incident from the lower layer at point P is refracted through the layer dh and touches the upper layer at point Q. Let θ be the angle of incidence with the normal drawn to the plane at P and $(\theta + d\theta)$ be the angle of refraction at$P$. In order to calculate the radius of curvature r, of the ray, draw angle bisectors at $P$and Q. Let them meet each other at O. From the inset in Fig. 4

$$\angle APQ = \pi - d\theta \tag{12}$$

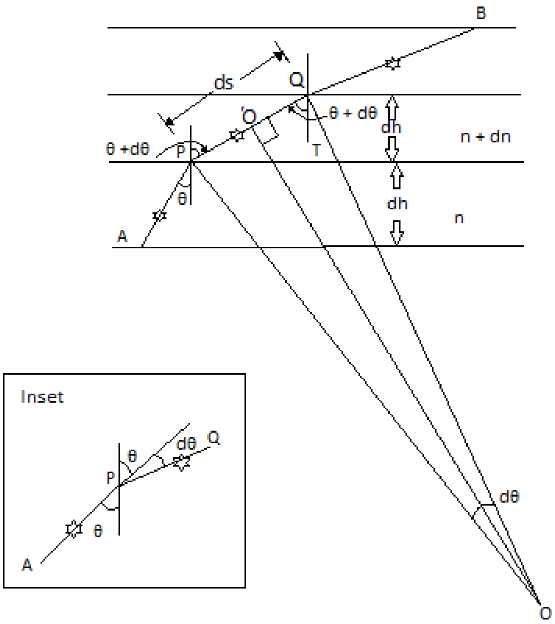

Figure 4. Wave propagation in a stratified medium

Since OP bisects $\angle APQ$

$$\angle OPQ = \frac{1}{2}\angle APQ = \frac{\pi}{2} - \frac{d\theta}{2} \tag{13}$$

Draw O'O perpendicular to PQ. In $\Delta PO'O$

$$\angle PO'O = \frac{\pi}{2} - \angle OPQ = \frac{d\theta}{2} \tag{14}$$

The angle subtended by the segment ds at O is

$$\angle OPQ = 2\angle PO'O = d\theta \tag{15}$$

and hence the length ds is given by

$$ds = rd \tag{16}$$

From ΔPQT

$$ds = \frac{dh}{\cos(\theta + d\theta)} \cong \frac{dh}{\cos\theta} \text{ for small } d\theta \tag{17}$$

Substituting the value of ds from Equation (16) into Equation (17)

$$r = \frac{ds}{d\theta} = \frac{dh}{cos\theta d\theta} \quad (18)$$

The law of refraction can now be applied to the two points P and Q

$$nsin\theta = (n + dn)\sin(\theta + d\theta) \quad (19)$$

Expanding the right hand side

$$nsin\theta = (n + dn)(sin\theta cosd\theta + cos\theta sind\theta) \quad (20)$$

which can be written as

$$cos\theta d\theta = -\frac{sin\theta dn}{n} \quad (21)$$

Substituting Equation (21) in Equation (18)

$$r = \frac{dh}{-sin\theta\frac{dn}{n}} = \frac{n}{sin\theta(-\frac{dn}{dh})} \quad (22)$$

The radius of curvature can also be written in terms of the grazing angle, φ.This is the angle that the ray makes with the horizontal and is related toθ by the equation

$$\theta = \frac{\pi}{2} - \varphi \quad (23)$$

The radius of curvature can be written in terms of φ as

$$r = \frac{n}{cos\varphi(-\frac{dn}{dh})} \quad (24)$$

Consider a ray launched with a low grazing angle,($\varphi \cong 0$), for which $cos\varphi \cong 1$. Since the refractive index is also very close to unity, the radius of curvature reduces to

$$r = \frac{1}{-\frac{dn}{dh}} \quad (25)$$

We can express the radius of curvature in terms of the refractivity gradient, dN/dh, as

$$r = \frac{10^6}{-\frac{dN}{dh}} \quad (26)$$

If this ray is propagating in the standard atmosphere described by the refractivity profile given by Equation (10), we have

$$\frac{dN}{dh} = -39/km \quad (27)$$

and the radius of curvature of the ray path is

$$r = \frac{1}{39\times10^{-6}} \cong 25641km \quad (28)$$

The ray droops towards the surface of the earth. If the refractivity does not change with the height, that is, $dN/dh = 0$, the radio wave does not undergo refraction hence it follows a straight line path. Figure 5 depicts the propagation of a radio wave launched at low grazing angles for various values of refractivity gradients (dN/dh). If the refractivity gradient is between 39/km and 0, the refraction of the electromagnetic wave is lower than that in the standard atmosphere and is known as sub-refraction. If the refractivity slope is less than that of the standard atmosphere, i.e., $dN/dh < -39/km$the wave is refracted more than that in a standard atmosphere and is known as super refraction. For $dN/dh = -157/km$, the radius of curvature of the ray is

$$r = \frac{1}{157 \times 10^{-6}} \cong 6370km \quad (29)$$

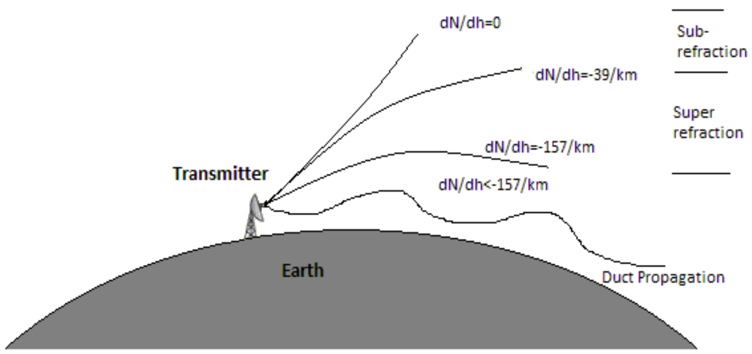

Figure 5. Wave propagation through a stratified medium above the spherical earth

which is equal to the radius of the earth, and hence a horizontally incident wave travels parallel to the surface of the earth. If $dN/dh = -157/km$, the radius of curvature of the ray is smaller than the radius of the earth. The ray can, therefore, touch the surface of the earth and get reflected from the surface. This is known as tropospheric duct propagation. The regions corresponding to sub-refraction, super refraction, and duct propagation are shown in Figure 6 on an $N - h$ plot. On a spherical earth, the maximum possible direct wave communication distance depends on the heights of the transmit and the receive antennas as well as the atmospheric conditions. For propagation calculations, a mean value of $r_0 = 6370km$ is chosen to be the radius of the earth. With $dN/dh = 0$, the radio waves travel along straight lines. Therefore, the maximum range is obtained when the straight line, joining the two antennas, grazes the surface of the earth.

ISSN:1791-4469

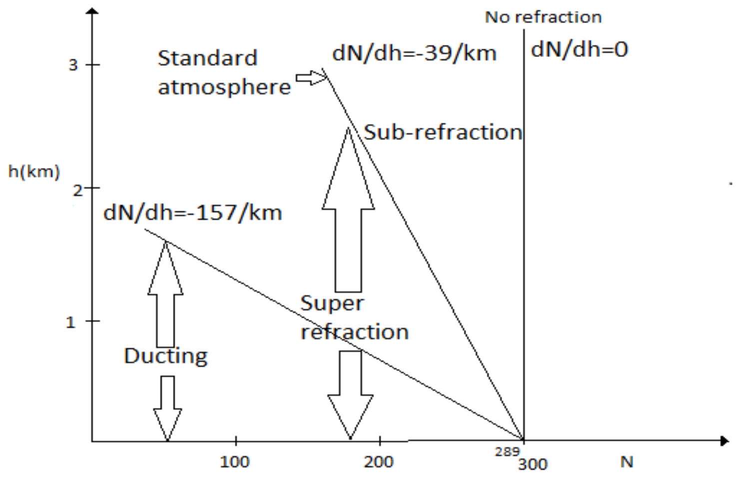

Figure 6. Depiction of regions of super refraction, sub-refraction, and duct propagation on an N-h plot.

3. PARABOLIC EQUATION MODELING

In this section we apply the finite-difference scheme of the Crank-Nicolson type on the Parabolic Equation [4] [7]. This technique permits the modelling with arbitrary boundaries.For the sake of initial simplicity, we choose to work with range vs height coordinates (x, z) assuming that the transverse coordinate y presents no variations on its own in the electromagnetic fields.The electric field E has only one non-zero component E_y, in horizontal polarization, while the magnetic field H has only one non-zero component H_y in vertical polarization. Thus we form a general variable field component ψ defined as $\psi(x,z) = E_y(x,z)$ for horizontal polarization or $\psi(x,z) = H_y(x,z)$ for vertical polarization. We assume a domain where the refractive index $n(x,z)$ has smooth variations, with suitable boundary conditions.We solve the wave equation where energy propagates at small angles from a preferred direction, called the paraxial direction. Let's choose the positive x-direction as the paraxial direction as shown in Figure 7.

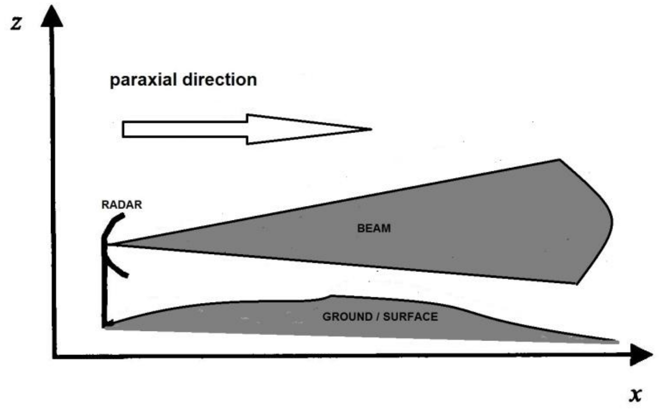

Figure 7. Tropospheric propagation of a radar beam in the x-paraxial direction [7]

For a homogeneous propagation medium with refractive index n, the field component ψ variations are modeled with a two-dimensional scalar wave equation:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial z^2} + k^2 n^2 \psi = 0 \tag{30}$$

where k is the wave number in vacuum. In reality, the refractive index varies with range x and height z, so the above equation is not accurate, but we use it as a first good approximation if the variations of n are slow compared to a wavelength. To further proceed in solving Equation (30) we introduce an auxiliary variable $u(x,z) = e^{-ikx}\psi(x,z)$ slowly varying in range for signal propagating at angles close to the paraxial direction, forming a new equation with convenient numerical properties :

$$\frac{\partial^2 u}{\partial z^2}(x,z) + 2ik\frac{\partial u}{\partial x}(x,z) + k^2(n^2(x,z)-1)u(x,z) = 0 \tag{31}$$

The solution for the above equation is the numerical values that u takes on a grid of (x,z) values placed over the domain. The u is assumed to be smooth almost everywhere such that it gets values for arbitrary x, z. The domain that we will work on is rectangular with x ranging from x_{min} to x_{max} and z ranging from 0 to N. Divide the intervals $[0, N]$and $[x_{min}, x_{max}]$ to J and N equally subintervals respectively. The length of these intervals is m in the z direction and h in the x direction. We denote with u_j^n the approximation at the grid point where $x_n = x_{min} + nh$, $z_j = jm$. To avoid the case that the numerical solution would be unstable we use the Crank-Nicholson algorithm that is unconditionally stable and also is second order accurate in both the x and z directions. The algorithm uses difference expressions for the partial derivatives that are centered around $x + \frac{h}{2}$ rather than around x. Thus the u, u_{zz} are averages of values of PE equation at time $n-1$ and n.

ISSN:1791-4469

$$u = \frac{u_j^{n-1} + u_j^n}{2} \tag{32}$$

$$\frac{\partial u}{\partial x} \approx \frac{u_j^n - u_j^{n-1}}{h} \tag{33}$$

$$\frac{\partial^2 u}{\partial z^2} \approx \frac{u_{j+1}^n + u_{j-1}^n - 2u_j^n}{m^2} \tag{34}$$

Substituting the above equations into the equation (30), we have

$$\frac{u_{j+1}^n + u_{j-1}^n - 2u_j^n}{m^2} + 2ik\frac{u_j^n - u_j^{n-1}}{h} + k^2\left(n^2\left(x_n, z_j\right) - 1\right)u_j^n = 0 \tag{35}$$

To above equations we suppose that

$$b = 4ik\frac{m^2}{h} \tag{36}$$

$$a_j^n = k^2\left(n^2\left(x_n, z_j\right) - 1\right)m^2 \tag{37}$$

So we have

$$u_j^n(-2 + b + a_j^n) + u_{j+1}^n + u_{j-1}^n = u_j^{n-1}(2 + b - a_j^n) - u_{j+1}^{n-1} - u_{j-1}^{n-1} \tag{38}$$

For $j = 1 \cdots N - 1$. The above system will be written in more convenient structure in matrix form,

$$\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & a_1^n & 1 & \cdots & 0 & 0 \\ 0 & 1 & a_2^n & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_{N-1}^n & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \begin{pmatrix} u_0^n \\ u_1^n \\ u_2^n \\ \vdots \\ u_N^n \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & b_1^n & -1 & \cdots & 0 & 0 \\ 0 & -1 & b_2^n & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & b_{N-1}^n & -1 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \begin{pmatrix} u_0^{n-1} \\ u_1^{n-1} \\ u_2^{n-1} \\ \vdots \\ u_N^{n-1} \end{pmatrix} \tag{39}$$

This system is easier to be solved by Gaussian elimination.

4. OTHER MODELING SCHEMES

Defense radar applications require predictions of the electromagnetic field variability in very large regions, up to several hundreds of km in range and several km in height. As Parabolic Equation (PE)integration times depend on frequency, propagation angles and domain size, calculations become prohibitively complex for such large domains. However, for most tropospheric problems, severe variations of the atmospheric refractive index never exist above a kilometre or so, and terrain irregularities are often limited to low altitudes resulting in perturbations of the propagation medium to be usually confined in height. Also since the refractive index of the air remains very close to unity, anomalous atmospheric refraction effects are limited to small propagation angles from the horizontal. Thus we reserve the powerful, but relatively slow PE modeling for field calculations at low heights and propagation angles, and to use faster methods of solving at other angles and heights of interest.

The Radio Physical Optics (RPO) model [3] combines ray optics and parabolic equation methods to accelerate computations for propagation over flat terrain. The domain is dividedinto four regions, as shown in Figure 7. In the flat Earth region, a simple two-ray model is used, assuming the rays propagate in straight lines. The idea here is that refractive effects are negligible for rays launched at angles of more than a few degrees from the horizontal (a threshold value of 5° is proposed in [3]), and there is hence no point in using more expensive ray-tracing algorithms. This modeling has been employed in the Advanced Propagation Model (APM) used by the software packet AREPS (Advanced Refraction Effects Prediction System) of SPAWAR, San Diego, California. The AREPS program computes and displays a number of electromagnetic (EM) system performance assessment tactical decision aids. These are radar probability of detection, electronic surveillance measure (ESM) vulnerability, HF to EHF communications, simultaneous radar detection and ESM vulnerability, and surface-search detection ranges.

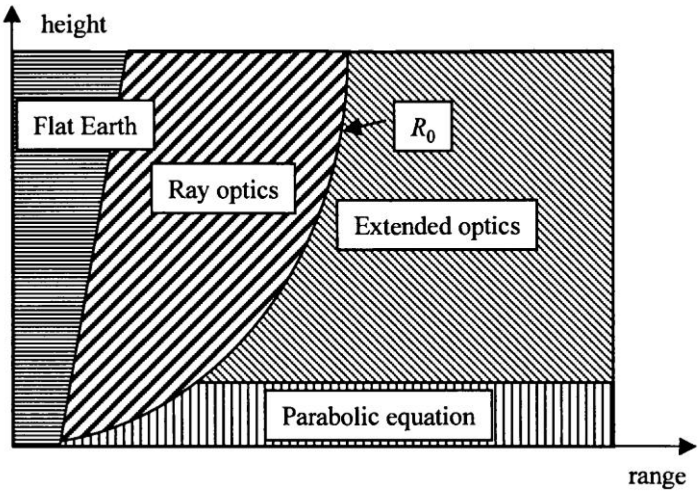

Figure 8. Regions used in AREPS [6].

5. CONCLUSIONS

Refraction or the bending of RF energy if not accounted for, can severely impact the performance of surface, littoral, naval or airborne radar and communication systems in a combat environment. These impacts influence operations, acquisition engineering, and prototype RF system testing. By combining modern numerical weather prediction models with RF system models, it is possible to create site and time-specific RF system performance forecasts that will ensure the survivability of any operating system in the future theatre of operations. Extensive numerical modeling leads to solutions that need to be validated with real time far field measurements of radar detection performance against target drones with known stereocopic RCS profiles.

REFERENCES

1. D. Griffiths, "Introduction to Elementary Particles", 1987 John Wiley & Sons, Inc.
2. S. D. Gunashekar et al, "Long-term statistics related to evaporation duct propagation of 2 GHz radio waves in the English Channel", http://onlinelibrary.wiley.com/doi/10.1029/2009RS004339/full.
3. H.V. Hitney, "Hybrid ray optics and parabolic equation methods for radar propagation modeling", Proceedings of Radar 92, IEE Conf. Pub., no. 365, pp. 58-61, 1992
4. S. A. Isaakidis and T. D. Xenos, "Parabolic Equation Solution of Tropospheric Wave Propagation Using FEM." Progress In Electromagnetics Research, PIER 49, 257–271, 2004.
5. ITU RECOMMENDATION ITU-R P.453-9 : The radio refractive index: its formula and refractivity data.
6. W. L. Patterson, "Advanced Refractive Effects Prediction System (AREPS)", SSC San Diego
7. M. Levy, Parabolic equation methods for electromagnetic wave propagation, IEE, 2000.