A Study On Radioactive Source Imaging By Using A Pixelated CdTe Radiation Detector

K. Zachariadou\textsuperscript{a,c}, K. Karafasoulis\textsuperscript{b,c}, S. Seferlis\textsuperscript{c}, I. Papadakis\textsuperscript{d}, D. Loukas\textsuperscript{d}, C. Lambropoulos\textsuperscript{e}, C. Potiriadis\textsuperscript{c},

\textit{a}. Technological Educational Institute of Piraeus, Department of Physics, Chemistry & Material Technology, P. Ralli & Thivon 250, 12244, Greece
\textit{b}. Hellenic Army Academy, 16673 Vari, Greece
\textit{c}. Greek Atomic Energy Commission, Patriarxou Grigoriou & Neapoleos, 15310 Athens, Greece
\textit{d}. National Centre for Scientific Research “Demokritos”, Institute of Nuclear Physics, 15310 Athens, Greece
\textit{e}. Technological Educational Institute of Chalkida, Psachna Evias 34400, Greece

Abstract. We present simulation studies on the ability of a proposed portable radiation detecting instrument to reconstruct images of radioactive sources. The instrument is a stack of ten CdTe layers placed 2 cm apart, consisting of 10000 pixels in a two-dimensional arrangement, occupying an area of 4cmx4cm. The image reconstruction is performed using LM-MELM, an imaging algorithm based on the Compton imaging technique. Point-like radioactive sources emitting gamma-rays in a broad energy range, located at various distances and orientations with respect to the detector’s symmetry axis have been simulated in order to estimate the ability of the instrument to reconstruct the source images. Results on the reconstructed image resolution are presented.

Keywords: Monte Carlo simulations, Semiconductor detectors, Gamma-ray spectroscopy, Compton camera.

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INTRODUCTION

Radiation detectors that use gamma-ray imaging technologies in order to identify radioactive sources are of great scientific interest because of their wide range of applications, including nuclear medicine, astrophysics, waste monitoring and counter terrorism. One of the best known such imaging technologies is the Compton imaging [1], based on the interactions of the emitted gamma-rays with the detector’s sensitive elements via the Compton scattering process. Although research in the Compton imaging technique’s applications have begun in the 1990s, the growing global interest for accurate detection of radioactive sources combined with the rapid advances in detector technologies (both in terms of material fabrication and electronics), have nowadays given great impetus to the research and development of Compton imaging detectors with enhanced detection capability.

In this framework, the main objective of the current research paper is to evaluate the imaging performance of a Compton imaging instrument under development (COCAE) [2].
COCAE consists of ten parallel planar layers made of pixelated Cadmium Telluride (CdTe) crystals occupying an area of 4cmx4cm, placed 2cm apart from each other. Each detector’s layer has 10,000 pixels arranged in two dimensions (100x100) bump-bonded on a two-dimensional array of silicon readout CMOS circuits. Both pixels and readout arrays are on top of an Al₂O₃ supporting printed circuit board layer.

The most important parameters in the design of a Compton imaging detector such as COCAE are the efficiency in detecting gamma-rays and the energy resolution, which affects the evaluation of the Compton scattering angle. COCAE is made of CdTe semiconductor crystals, thus it is expected to achieve an enhanced detection efficiency compared to Germanium (Ge) and Sodium Iodide (NaI) detectors, due to the higher atomic number of Cd and Te, resulting into a higher absorption of gamma-rays via the photoelectric effect. In order to achieve even better efficiency, a thick CdTe detector of several mm would be needed but such an increase of the crystal’s thickness would deteriorate the detector energy resolution (due to the effect of incomplete charge collection of CdTe semiconductors). To bypass this restriction, COCAE instrument has been designed as a system of many thin stacked CdTe crystals instead of one thick mono-crystal. As to the energy resolution parameter, the challenge for COCAE is to achieve a high energy resolution without the need of cryogenics (CdTe semiconductors can be operated at room temperature due to their high energy bandwidth), which is important when considering a portable instrument.

In order to study its performance, the COCAE instrument is modelled (Figure 1) by an open-source object-oriented software library (MEGAlib [3] which provides an interface to Geant4 [4], a toolkit that simulates the passage of particles through matter. The radioactive sources are modelled as point-like mono-energetic gamma sources located at various distances and orientations with respect to the detector, emitting gamma-rays having energies in the range from 100keV to 2000keV. The simulated energy depositions are blurred according to Gaussian distributions with a FWHM that varies from 3.5% at energies around 100keV down to 1% at energies above 662keV, assumed to be in accordance with realistic energy measurements. At least one billion of gamma rays are emitted from the radioactive sources for each simulation condition.

![Figure 1](image-url)  
**FIGURE 1.** A simulated gamma-ray interacts with the COCAE detector. Three energy depositions are recorded.

Important performance parameters of the COCAE instrument such as its detecting efficiency and angular resolution have been studied by Monte Carlo in our previous work [5] as well as various techniques for the determination of the correct sequence of multiple Compton scattering interactions of the gamma rays with the detector’s sensitive materials, in a wide range of incident gamma-ray energies [6], [7], [8].

The current research work aims to study the ability of the COCAE instrument to reconstruct the image of point-like radioactive sources located at different orientations and source-to-detector distances, emitting gamma-rays in a wide range of energies. Described in the following
sections is the image reconstruction algorithm used as well as the resolution of the reconstructed images.

**SOURCE IMAGE RECONSTRUCTION**

The process of Compton imaging refers to the generation of the image of a radioactive source by using the recorded information of the energy depositions and the positions of the interactions of many incident gamma rays with the sensitive materials of the detector. There are several algorithms that yield Compton images. For the current research work the selected imaging algorithm is the List Mode Maximum Likelihood Expectation Maximization (LM-MLEM). This technique has been originally developed for medical imaging [9] and has been wide-spread in the field of Compton imaging [6].

LM-MLEM is an iterative algorithm that converges to the source image with the highest likelihood of having produced the recorded data. It is a list mode based algorithm (LM); the data fed to the algorithm is a mere list of events (energy depositions and positions of interactions of the incident gamma rays via the Compton scattering). The list mode methods are appealing in image reconstruction because the total number of data in the list is significantly smaller than the number of possible combination of position and energy measurements.

The imaging algorithm starts the reconstruction of the image for each event at a time by using the back projection method illustrated in Figure 2, which is an algorithm that projects the Compton cone of each event onto an imaging plane. The Compton cone refers to the well known Compton scattering process: When an emitted gamma-ray interacts with the detecting materials via the Compton scattering effect, a recoil electron and a scattered photon are created in a such way so both the energy and the momentum of the scattering is conserved. The energy ($E_e$) and the position ($\hat{r}_1$) of the recoil electron can be quickly measured while the scattered photon ideally deposits all its energy ($E_g$) in the detecting materials in a series of one or more interactions before it is finally absorbed via a photoelectric interaction. (The position of an interaction is assumed to be the center of a CdTe pixel). The scattering angle ($\theta$) is related to the energy depositions both of the recoil electron and of the scattered photon via the well known Compton formula:

$$\cos \theta = 1 - m_0c^2 \left( \frac{1}{E_g} - \frac{1}{E_g + E_e} \right)$$

where $m_0c^2$ is the rest energy of the electron.

Moreover, the scattering angle is geometrically related to the direction ($\hat{r}_0$) of the incident gamma-ray via the formula:

$$\cos \theta = \hat{r}_0 \cdot \hat{u}$$

where $\hat{u} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$

Thus, by recording the positions of the interactions ($\vec{r}_1$, $\vec{r}_2$) and the energy depositions ($E_e$, $E_g$), the incident direction ($\vec{r}_0$) of the primary gamma-ray is constrained to lie on a cone (Compton cone). The Compton cone’s central axis is defined by the vector $u$ given by equation (2) connecting the two interactions whereas the cone’s opening angle is equal to the Compton scattering angle defined by equation (1). The apex of the Compton cone is located at the CdTe crystal pixel in which the gamma-ray have scattered.

Since the incident gamma ray could have originated from any point on the surface of the Compton cone, during the image reconstruction, the back projection algorithm samples randomly the azimuthal angle ($\varphi$) over the full range $[0,2\pi]$. Then, each of the vectors
corresponding to a specific value of $\varphi$ directed towards to the apex of the cone is projected onto multiple planes at different source-to-detector distances ($z$). For any given projection imaging plane, the pixels intersected by the cone form a circle scribed onto the projection plane.

**FIGURE 2.** The Back projection imaging algorithm

All back projected Compton circles (each one corresponding to a different incident gamma-ray emitted by the radioactive source) intersect at a common point that determines the source location. In principle three Compton cones should be enough to reconstruct the image of a point-like radioactive source but in practice (due to measurement errors and to incomplete absorption of the scattered photon) a large number of reconstructed Compton cones are needed to derive the source location accurately (Figure 3).

**FIGURE 3.** The back projected image process: as the number of events and the number of iterations increase (from top to bottom), the source image converges.
The back projected image serves as the initial estimation of the image to start the iteration procedure of the LM-MLEM algorithm. For each pixel in the image all the events having a Compton cone that touched that pixel are recorded and the pixel’s sensitivity is calculated, representing the probability that a gamma ray originated by the pixel is detected anywhere in the detector. This information is fed to the LM-MLEM algorithm that uses an iterative reconstruction equation for calculating the amplitude of each pixel of the image in order to find the reconstructed image distribution with the highest likelihood of having produced the recorded data.

Figure 4 depicts the reconstructed image for the case of an 800keV point-like radioactive source using 50 iterations of the LM-MLEM imaging algorithm. The simulated source is located 50 cm from the detector’s center at inclination angle \( \theta = 26.56^\circ \) and azimuth angle \( \varphi = 0^\circ \) (where \( \theta = 0^\circ \) corresponds to the detector’s symmetry axis), in spherical coordinates.

**FIGURE 4.** Reconstructed image using 50 iterations of LM-MLEM imaging algorithm, for the case of \( z \) simulated point-like radioactive source (800 keV) located at \( (\theta = 26.56^\circ, \varphi = 0^\circ) \).

**RECONSTRUCTED IMAGE RESOLUTION**

The ability of the COCAE instrument to estimate the orientation of radioactive sources depends largely on its resolution of reconstructing radioactive sources’ images. The reconstructed image resolution is defined as the combined FWHM of the azimuth (\( \varphi \)) and inclination (\( \theta \)) profiles of the source’s reconstructed image measured in steradian (sr).

We have studied the reconstructed image resolution of the COCAE instrument by considering two case conditions: point-like radioactive sources located a) on the detector’s symmetry axis \( (z) \) and b) off the detector’s symmetry axis.

For the evaluation of the reconstructed image resolution for various gamma-ray energies, source-to-detector distances and orientations, the same number of interactions of incident gamma rays with the detector has been assumed. This assumption can be achieved experimentally by increasing the acquisition time as a function of the source-to-detector distance.

Shown in Figure 5a and 5b are the azimuth and inclination distributions respectively (measured in degrees), for the case of an 800keV point-like radioactive source located 50 cm from the detector’s center at azimuth angle \( \theta = 26.56^\circ \) and inclination angle \( \varphi = 0^\circ \).
Radioactive sources located on the detector’s symmetry axis

For the case of on-axis radioactive sources, our studies have been performed by simulating radioactive sources emitting gamma rays in an energy range from 100keV to 2MeV located at distances up to 2m from the detector’s centre.

Presented in Figure 6 is the reconstructed image resolution (measured in steradian (sr) as a function of the source-to-detector distance for three cases of point-like radioactive sources emitting 400keV, 1000keV and 2000keV gamma rays. It can be noticed that the reconstructed image resolution increases as the source-to-detector distance is reduced; it varies from less than 2.5x10⁻³ sr (for source-to-detector distances ~50cm) down to about 0.5x10⁻³ sr (for point-like sources located at distances greater than ~1m).
FIGURE 6. Reconstructed image resolution for point-like 400 keV, 1000 keV and 2000 keV radioactive sources located on the detector’s symmetry axis, as a function of source-to-detector distance.

Furthermore, it can be seen from Figure 7 that for an arbitrary source-to-detector distance (z=80cm) and for point-like radioactive sources emitting gamma rays with energies in the range from 400 keV to 2000 keV, the reconstructed image resolution is less than $1 \times 10^{-3}$ sr.

FIGURE 7. Reconstructed image resolution for point like radioactive sources located on the detector’s symmetry axis as a function of the incident gamma ray energy, for an arbitrary source-to-detector distance (z=80cm).

Moreover, we have studied the dependence of the COCAE’s image resolution on the number of the reconstructed events. (It has to be noticed that for the reconstruction of the source’s image only a fraction of the reconstructed events is used, corresponding only to those events that interact with the COCAE’s detecting elements via the Compton scattering process). Figure 8a and 8b show the azimuth ($\phi$) and the inclination ($\theta$) coordinate respectively of the reconstructed image, as a function of the number of reconstructed events. The simulated point-like radioactive sources emit gamma rays in the energy range from 400 keV to 1250 keV and they are located at an arbitrary source-to-detector distance (z=120 cm), on the detector’s symmetry axis ($\phi=180^\circ$, $\theta=90^\circ$). The solid line in Figure 8a and 8b represents the real azimuth and inclination coordinate of the simulated sources respectively, whereas the error bars correspond to the estimated FWHM of the image distributions, measured in degrees (deg). It
can be seen that a minimum number of about $5 \times 10^3$ reconstructed events is required for a successful image reconstruction.

![Graphs showing azimuth and inclination image coordinates vs. number of reconstructed events.]

**FIGURE 8:** The azimuth (top) and inclination (bottom) image coordinate of point-like radioactive sources located on the detector’s symmetry axis (solid line), as a function of the number of reconstructed events.

**Radioactive sources located off the detector’s symmetry axis**

For the case of point-like radioactive sources located off the detector’s symmetry axis (defined at $\phi=0^\circ, \theta=0^\circ$) we have performed Monte Carlo studies, by simulating radioactive sources emitting gamma rays in the energy range from 100keV to 2MeV. The sources are located at 50 cm from the detector’s center at azimuth angle $\phi=0^\circ$ and inclination angles up to $90^\circ$.

Figure 9 illustrates the evaluated reconstructed image resolution as a function of the incident gamma ray energy, for the case of point-like radioactive sources located at various inclination angles whereas Figure 10 shows the reconstructed imaging resolution versus the inclination angle, for 600keV, 1000keV and 2000keV radioactive sources.

It can be noticed that the reconstructed image resolution is worse compared to the case of on-axis sources being less than $\sim 4 \times 10^{-3}$sr, for point-like radioactive sources emitting gamma rays with energies from 600keV to 2000keV.
CONCLUDING REMARKS

The reconstructed image resolution of a portable pixelated CdTe detector (COCAE) has been studied by applying the LM-MLEM imaging algorithm on a large number of simulated gamma rays. The radioactive sources are modelled as point-like mono-energetic gamma sources located at various distances and orientations with respect to the detector’s symmetry axis, emitting gamma-rays having energies in a broad energy range from 100keV to 2000keV.

Our studies have shown that for radioactive sources placed on the detector’s symmetry axis at least five thousand reconstructed events are needed for a successful reconstruction of the source’s image. Using fifty iterations of the LM-MLEM imaging algorithm, the reconstructed image resolution has been estimated to be less than \(\sim 2.5 \times 10^{-3}\) sr (for source-to-detector distances \(\sim 50\)cm) down to \(\sim 0.5 \times 10^{-3}\) sr (for point-like sources located at distances greater than \(\sim 1\)m).
Moreover, simulation studies performed for radioactive sources placed off the detector’s symmetry axis have showed that the ability of the instrument to reconstruct the image is worse than in the case of on-axis sources, being less than about $4 \times 10^{-3} \text{ sr}$.

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REFERENCES