# Maximum Effective Bit Rate Estimation for Wireless Optical Communication Links with Time-Diversity Over Strong Turbulence Channels

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**Abstract.** The free space optical communication systems exhibit significant research and commercial interest in the last few years due to their capability of achieving optical wireless communications with high and secure data rate transmission and low installation and operational cost, without need of licensing. On the other hand, the unstable conditions of the atmosphere, which is the propagation path of the laser beam that carries the information signal, and especially the atmospheric turbulence effect, affects mainly its intensity and causes random irradiance fluctuations at the receiver's end. This phenomenon causes a significant reduction in the system's availability and performance (BER). In order to overcome this reduction, many methods have been studied, from which the diversity techniques are a very efficient solution. In this work, we investigate a time-diversity scheme for free space optical channels under strong turbulence conditions modeled with the K distribution. We concentrate on the estimation of the maximum effective bit rate of the link, which is limited by the finite capacity of the optical channel, as well as on the multiple transmission of the same part of the information signal in order to achieve much better performance characteristics using the time diversity scheme. Thus, we extract closed form mathematical expressions for the estimation of the maximum effective bit rate, for both, fast and slow, fading statistics. Finally, we present numerical results for many practical cases.

**Keywords:** K-distribution, Time diversity, Free Space Optical Communication Systems, Atmospheric Turbulence, Maximum Effective Bit Rate, Channel Capacity.
**PACS:** 42.79.Sz, 92.60.hk.

## INTRODUCTION

Free space optical (FSO) communication systems enable the communication between two points with a laser beam using the atmosphere as a channel. This kind of wireless communication has many advantages compared to other methods, such as high speed of data transfer (bit-rate), no need of license to transmit in this band of frequencies, low installation and operation cost. On the other hand, the unstable conditions of the atmosphere and especially the atmospheric turbulence effect, affects the intensity of the laser beam causing random irradiance fluctuations at the receiver's end. So, the atmosphere is a channel with randomly time varying characteristics [1]-[8].

Thus, it is obvious that the atmospheric turbulence is a mitigation factor for the performance and reliability of the FSO link, due to the induced scintillation effect [4]-[10]. In order to overcome this influence, many techniques have been investigated. The diversity schemes represent one of them [11]-[19]. In this technique, the information signal is sent more than once, using different spatial paths, i.e. spatial diversity, different wavelengths, i.e. wavelength diversity, different polarizations, i.e. polarization diversity, or different time slots, i.e. time diversity [16], [19]. Thus, each copy of the information signal propagates through a channel with different characteristics and obviously the probability of error is decreasing as well as the probability of outage of the FSO system.

In this work, we are studying an FSO communication link over atmospheric turbulence channel by assuming strong turbulence conditions modeled with the K-distribution [20]-[22], with a time diversity scheme. In this configuration, the main advantage is the requirement of one and only trans-receiver pair which is sending the same information signal more than once in different time slots. On the other hand, taking into account that the channel's capacity is finite, this technique reduces the actual effective maximum bit rate of the link. Thus, we present mathematical expressions for the estimation of this limit for both, slow and fast, fading statistics of the wireless connection.

The rest of this work is organized as follows: in Section II, we introduce the considered FSO system and channel model for the time diversity scheme. In Section III, we derive closed-form mathematical expressions for the estimation of the maximum effective bit rate of the FSO link for slow and fast fading statistics, while in Section IV, we present numerical results for the system's reliability and performance for various link's parameters. Final conclusions are presented in Section V.

## THE FSO CHANNEL MODEL WITH TIME DIVERSITY

As mentioned above, for the FSO links with time diversity, only a single pair of transmitter and receiver is needed and the information signal is sent more than once using different time slots. Thus, this diversity scheme can be emulated with an FSO communication system which is using one transmitter that transmits *M* copies of the signal at *M* different time-slots and one receiver for these *M* copies. Hence, this operation is equivalent to the combined operation of one transmitter transmitting through *M* channel/branches and *M* receivers at the receiving end, similar with the Single Input Multiple Output (SIMO) communication systems [19].

The system's input is binary and the output continuous intensity modulation/direct detection (IM/DD), with On-Off keying (OOK) modulation. Moreover, the channel is characterized as stationary and memoryless with independent and identically distributed intensity (i.i.d) fading statistics, with additive white Gaussian noise (AWGN). In this case, the statistical channel model can be expressed as in [14], [15], [23]:

$$r_m = s_m x + n = \eta x I_m + n, \qquad m = 1, \ldots, M \tag{10}$$

where $y_m$ is the signal at the receiver for each one of the *M* copies of the information signal, $s_m = \eta_m I_m$ is the instantaneous intensity gain, $\eta$ is the effective photo-current conversion ratio of the receiver, $I_m$ is the normalized irradiance arrived in each receiver, *x* is the modulated signal (taking the binary values "0" or "1"), and *n* represents the additive white Gaussian noise (AWGN) with zero mean and variance equal to $N_0/2$, [24], [25].

For strong atmospheric turbulence conditions modeled with the K distribution, the probability density function (pdf) of this model is given in [21], as:

$$f_{I_m}(I_m)=\frac{2(a_m)^{\frac{1+a_m}{2}}}{\Gamma(a_m)}I_m^{\frac{a_m-1}{2}}K_{a_m-1}\left(2\sqrt{a_m I_m}\right) \tag{11}$$

where $K_v(.)$ is the modified Bessel function of the second kind of order $v$ and $\Gamma(.)$ is the gamma function. In addition, $a_m$ represents a parameter that is generally associated with the number of scatterers forming the random component of the optical field [20]-[22]. By integrating (2), we conclude the corresponding cumulative distribution function (cdf), which has the following expression [21]:

$$F_{I_m}(I_m)=\frac{(a_m I_m)^{\frac{1+a_m}{2}}}{\Gamma(a_m)}G_{1,3}^{2,1}\left(b_m I_m\left|\begin{matrix}\frac{1-a_m}{2}\\ \frac{1-a_m}{2},\frac{a_m-1}{2},-\frac{1+a_m}{2}\end{matrix}\right.\right) \tag{12}$$

Next, we define the instantaneous electrical signal-to-noise ratio (SNR) as $\xi_m=(\eta I_m)^2/N_0=s_m^2/N_0$, [25], and the average electrical SNR as $\mu_m=(\eta E[I_m])^2/N_0$, [25], [26], with *E[.]* being the expected value of the normalized irradiance of the $m_{th}$ copy of the information signal $I_m$. Thus, the pdf and the cdf of $\xi_m$, for the K distribution, are given as [21]:

$$f_{\xi_m}(\xi_m)=\frac{(a_m)^{\frac{1+a_m}{2}}}{\Gamma(a_m)}\frac{\xi_m^{\frac{1+a_m}{4}-1}}{\mu_m^{\frac{1+a_m}{4}}}K_{a_m-1}\left(\sqrt[4]{\frac{16a_m^2\xi_m}{\mu_m}}\right) \tag{13}$$

and

$$F_{\xi_m}(\xi_m)=\frac{(a_m)^{\frac{1+a_m}{2}}}{\Gamma(a_m)}\left(\frac{\xi_m}{\mu_m}\right)^{\frac{1+a_m}{4}}G_{1,3}^{2,1}\left(\sqrt{\frac{a_m^2\xi_m}{\mu_m}}\left|\begin{matrix}\frac{1-a_m}{2}\\ \frac{1-a_m}{2},\frac{a_m-1}{2},-\frac{1+a_m}{2}\end{matrix}\right.\right) \tag{14}$$

where $G_{p,q}^{m,n}[\cdot]$ is the Meijer G-function, [27].

For a time diversity scheme, due to the fact that we use one pair of transmitter and receiver and consequently the spatial propagation path is the same, we can assume that the value of $a_m$, of Eqs (4) and (5), as well the average electrical SNR, $\mu_m$, remain invariable for all the $M$ copies of the transmitted signal [19]. Hence, the pdf and the cdf for $\xi_m$, is obtained from (4) and (5) by assuming that $a=a_1=a_2=\ldots=a_M$ and $\mu=\mu_1=\mu_2=\ldots=\mu_M$, [19] and we have the following mathematical forms, [21]:

$$f_{\xi_m}(\xi_m)=\frac{a^{\frac{1+a}{2}}}{\Gamma(a)}\frac{\xi_m^{\frac{a-3}{4}}}{\mu^{\frac{1+a}{4}}}K_{a-1}\left(\sqrt[4]{\frac{16a^2\xi_m}{\mu}}\right) \tag{15}$$

and

$$F_{\xi_m}(\xi_m)=\frac{a^{\frac{1+a}{2}}}{\Gamma(a)}\left(\frac{\xi_m}{\mu}\right)^{\frac{1+a_m}{4}}G_{1,3}^{2,1}\left(\sqrt{\frac{a^2\xi_m}{\mu}}\,\middle|\,\begin{matrix}\frac{1-a}{2}\\ \frac{1-a}{2},\frac{a-1}{2},-\frac{1+a}{2}\end{matrix}\right) \tag{16}$$

## MAXIMUM EFFECTIVE BIT RATE OF THE LINK WITH TIME DIVERSITY

It is well known that the time diversity schemes decrease the outage probability of the FSO links, as well as their probability of error, and as a result, increase their availability, reliability and performance [16], [19]. On the other hand, due to the fact that the time diversity schemes are using one trans-receiver pair and the capacity of the channel is finite, the main disadvantage of this technique is that the multiple transmissions of the same parts of the information signal, decrease the maximum practical rate of data transmission. In this section, we will use significant mathematical expressions for FSO links without diversity, in order to investigate the maximum effective bit rate of the link with time diversity, $R'$. Obviously, this quantity is lower than the maximum bit rate of the link, $R$, due to the fact that it represents the practical rate of bit transmission, taking into account that with this technique, the same bit, occupies the trans-receiver $M$ different time slots. The maximum bit rate, $R$, that can be transmitted through a single channel with fast fading statistics is determined by its average capacity, $C_{av}$, [19], [28]-[31], which represents the practically achievable error-free bit-rate transmission. While, for the case of slow fading statistics [19], [22], [32], [33], the value of $R$ is estimated by its outage capacity, $C_{out}$, which stands for a metric that equals to the guaranteed capacity for channel realizations with a probability of (1-r), i.e. $\Pr[C<C_{out}]=r$, [19], [22], [32], [33]. Thus, taking into account that the time diversity scheme uses only one FSO channel, the average (outage) channel capacity of the whole scheme, coincides with $R$ for the case of fast (slow) fading statistics [19].

Hence, we can derive a new mathematical expression for $R$, using the expressions obtained in [21], [22]. Thus, for $M$ copies of the same bits of the information signal, the maximum effective bit rate of the time diversity FSO link, could be defined as [12], [19]:

$$R'=\frac{R}{M}\equiv\frac{C}{M} \tag{17}$$

where $C$ equals either to $C_{av}$ or to $C_{out}$, according to the channel's fading statistics. Thus, by substituting in Eq. (8) the results obtained in Refs [21] and [22] for average and outage channel's capacity, respectively, we conclude to the following closed form mathematical expressions for the estimation of the maximum effective bit rate, $R'_{av}$ and $R'_{out}$, that are derived for both types of fading statistics, respectively:

$$R'_{av}=\frac{a^{\frac{1+a}{2}}B}{4\pi M\ln(2)\Gamma(a)\mu^{\frac{\alpha+1}{4}}}G^{6,1}_{2,6}\left(\frac{a^{2}}{16\mu}\left|\begin{matrix}-\frac{1+a}{4},\frac{3-a}{4}\\ \frac{a-1}{2},\frac{a+1}{4},\frac{1-a}{4},\frac{3-a}{4},-\frac{1+a}{4},-\frac{1+a}{4}\end{matrix}\right.\right) \tag{18}$$

and

$$r=\frac{\left(2^{MR'out/B}-1\right)^{\frac{a+1}{4}}}{\left(\sqrt{\mu}/\alpha\right)^{\frac{\alpha+1}{4}}\Gamma(a)}G^{2,1}_{1,3}\left(a\sqrt{\frac{2^{MR'out/B}-1}{\mu}}\left|\begin{matrix}\frac{1-a}{2}\\ \frac{1-a}{2},\frac{a-1}{2},-\frac{1+a}{2}\end{matrix}\right.\right) \tag{19}$$

with *B* being the channel's bandwidth, while the quantity (1-*r*) represents the probability of channel realizations where the capacity, $C_{out}$, is guaranteed.

## NUMERICAL RESULTS

Using the expressions (9) and (10), we are able to present the maximum effective bit rate and the outage capacity as a function of the average electrical SNR, for the FSO link that uses time diversity scheme, for different parameter values. More specifically, we have the case without time diversity, *M*=1 and two cases with diversity, i.e. *M*=3 and *M*=5, for two values of parameter *α*, i.e. 1 and 3.

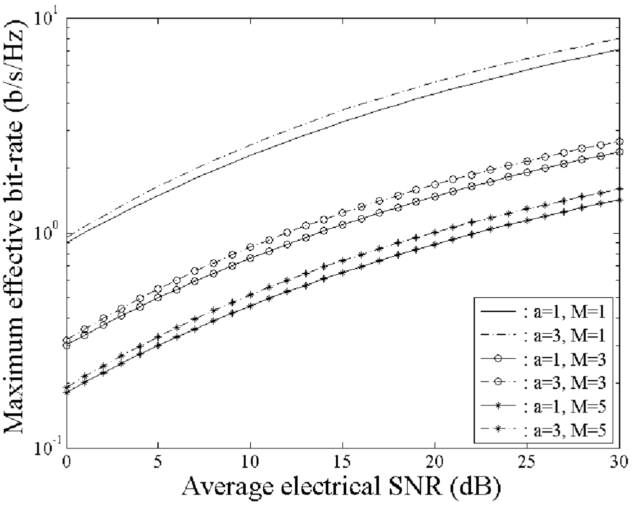

**FIGURE 1.** Normalized maximum effective bit rate, $R_{av}/B$, of an FSO link with time diversity versus the average electrical SNR, *μ*, for various values of *M* and *α*.

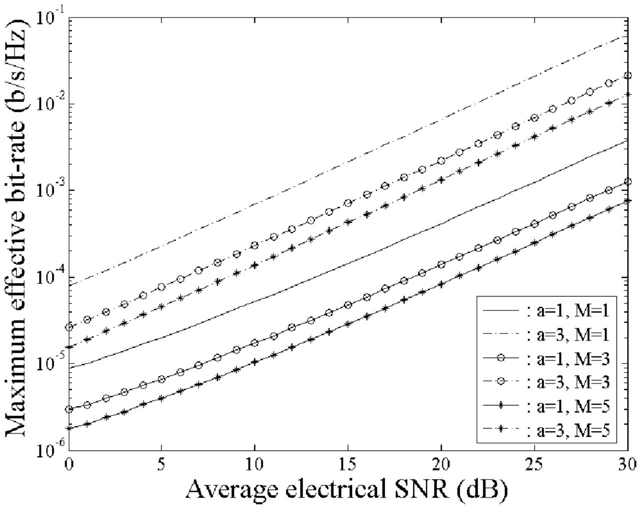

**FIGURE 2.** Maximum effective bit rate, $R_{out}/B$, of an FSO link with time diversity versus the average electrical SNR, $\mu$, for various values of $M$ and $\alpha$.

In Fig. (1), we present the results for the maximum effective bit rate. We can see that as the parameter $\alpha$ increases, the bit-rate of the channel increases too, as it was expected. Moreover, in Fig. (1) it is clear that the use of time diversity technique decreases the maximum effective bit rate of the channel of the FSO link. This result in the time diversity scheme is due to the fact that the same information signal must be sent multiple times at different time slots.

In Fig. (2), we present the corresponding results for the maximum effective bit – rate, obtained from the outage capacity with r=0.01 (i.e. percentage rate 1%) which is a typical value in order to estimate the outage capacity.

## CONCLUSIONS

In this work we estimated the maximum effective bit rate that an FSO communication link with time diversity, can achieve, under strong atmospheric turbulence conditions, modeled with the K distribution. This bit rate represents the practical maximum bit rate that such a system can attain due the fact that these systems are using one and only trans-receiver pair and the channel's capacity is finite. We derive closed form mathematical expressions for the estimation of this metric for both cases of fading statistics, i.e. fast and slow. Through the obtained expressions, we presented various numerical results for the maximum effective bit rate, for different values of SNR arriving at the receiver and the time diversity parameter. These results clearly demonstrate that although time diversity techniques, achieve improved availability and performance (BER) of an FSO link, they lead to a decrease of the maximum bit rate.

## REFERENCES

1. D. Keddar, and S. Arnon, "Urban Optical Wireless Communication Networks: the Main Challenges and Possible Solutions," *IEEE Opt. Com.,* pp. S1-S7, 2004.
2. S. Arnon, "Optical Wireless Communications," *Encycl. Opt. Eng.*, Marcel Dekker Inc. New York, pp. 1866-1886, 2003.

3. H. Henniger, and O. Wilfert, “An Introduction to Free-Space Optical Communications,” *Radioengineering*, vol. 19, iss. 2, pp. 203-212, 2010.
4. A.K. Majumdar, “Free-Space Laser Communication Performance in the Atmospheric Channel,” *J. Opt. Fiber Commun. Rep.*, vol. 2, pp. 345-396, 2005.
5. W.O. Popoola, and Z. Ghassemlooy, “BPSK Subcarrier Intensity Modulated Free-Space Optical Communications in Atmospheric Turbulence,” *IEEE/OSA Journal of Lightwave Technology*, vol. 27, iss. 5-8, pp. 967-973, 2009.
6. L.C. Andrews, R.L. Phillips, C.Y. Hopen, Laser Beam Scintillation with Applications, SPIE Optical Engineering Press, (2001).
7. L.C. Andrews, M.A. Al-Habash, C.Y. Hopen, and R.L. Phillips, Theory of Optical Scintillation: Gaussian Beam Wave Model, Waves in Random Media, 11 (2001) 271-291.
8. Z. Ghassemlooy and W.O. Popoola, "Terrestrial Free-Space Optical Communications", book chapter in "Mobile and Wireless Communications: Network Layer and Circuit Level Design", Salma Ait Fares and Fumiyuki Adachi (Ed.), ISBN: 978-953-307-042-1, InTech, 2010.
9. M.H. Mahdieh and M. Pournoury, Atmospheric Turbulence and Numerical Evaluation of Bit Error Rate (BER) in Free-Space Communication, Optics and Laser Technology, 42, 1 (2010) 55-60.
10. W.O. Popoola, Z. Ghassemlooy, C.G. Lee and A.C. Boucouvalas, “Scintillation effect on intensity modulated laser communication systems - a laboratory demonstration”, *Optics & Laser Technology*, vol. 42, pp. 682–692, 2010.
11. R. Rachmani, and S. Arnon, “Wavelength Diversity in Turbulence Channels for Sensor Networks,” *IEEE 26th Convention of Electrical and Electronics Engineers in Israel, IEEEI 2010*, art. no. 5661946, pp. 915-918, 2010.
12. H.E. Nistazakis and G.S. Tombras, “On the use of Wavelength and Time Diversity in Optical Wireless Communication Systems over Gamma-Gamma Turbulence Channels”, *Elsevier, Journal of Optics & Laser Technology*, Vol. 44, Iss. 7, pp. 2088-2094, 2012.
13. S.M. Navidpour, M. Uysal, and M. Kavehrad, “BER Performance of Free Space Optical Transmission with Spatial Diversity,” *IEEE Trans. Wireless Commun.*, vol. 6, iss. 8, pp. 2813-2819, 2007.
14. T.A. Tsiftsis, H.G. Sandalidis, G.K. Karagiannidis, and M. Uysal, “Optical Wireless Links with Spatial Diversity Over Strong Atmospheric Turbulence Channels,” *IEEE Transactions on Wireless Communications*, vol. 8, iss. 2, art. no. 4786457, pp. 951-957, 2009.
15. A.N. Stassinakis, H.E. Nistazakis, and G.S. Tombras, “Comparative Performance Study of One or Multiple Receivers Schemes for FSO Links Over Gamma Gamma Turbulence Channels”, *Taylor & Francis Group, Journal of Modern Optics*, Vol. 59, Iss. 11, pp. 1023-1031, 2012.
16. F. Xu, A. Khalighi, P. Caussé, and S. Bourennane, “Channel Coding and Time-Diversity for Optical Wireless Links,” *Optics Express*, vol. 17, iss. 2, pp. 872-887, 2009.
17. E. Wainright, H.H. Refai, and J.J. Sluss Jr., “Wavelength Diversity in Free-Space Optics to Alleviate Fog Effects”, *Proceedings of SPIE - The International Society for Optical Engineering*, vol. 5712, art. no. 16, pp. 110-118, 2005.
18. E.J. Shin, V.W.S. Chan, Optical Communication Over the Turbulent Atmospheric Channel using Spatial Diversity, Conference Record / IEEE Global Telecommunications Conference 3, (2002) 2055-2060.
19. H.E. Nistazakis, “A Time-Diversity Scheme for Wireless Optical Links Over Exponentially Modeled Turbulence Channels”, *Elsevier, Optik -International Journal for Light and Electron Optics*, in press.
20. M. Uysal, S.M. Navidpour and J. Li, “Error Rate Performance of Coded Free Space Optical Links Over Strong Turbulence Channels", IEEE Communications Letters, Vol. 8, no 10, 2004.
21. H.G. Sandalidis, T.A. Tsiftsis, “Outage probability and ergodic capacity of free-space optical links over strong turbulence”, *Electronics Lett*ers, Vol. 44, no. 1, p.p. 46 – 47, 2008.
22. H.E. Nistazakis, A.D. Tsigopoulos, M.P. Hanias, C.D. Psychogios, D. Marinos, C. Aidinis, and G.S. Tombras, Estimation of Outage Capacity for Free Space Optical Links Over I-K and K Turbulent Channels, Radioengineering, 20, 2 (2011) 493-498.
23. R. Purvinskis, D. Giggenbach, H. Henniger, N. Perlot, and F. David, “Multiple Wavelength Free-Space Laser Communications,” *Proceedings of SPIE - The International Society for Optical Engineering*, vol. 4975, pp. 12-19, 2003.
24. H.E. Nistazakis, V.D. Assimakopoulos, and G.S. Tombras, “Performance Estimation of Free Space Optical Links Over Negative Exponential Atmospheric Turbulence Channels,” *Elsevier OPTIK*, vol. 122, pp. 2191-2194, 2011.